Relativistic Exercises

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1 Gyroradius of a TeV Proton

What is gyroradius (in parsecs) of a 1 TeV proton in a galactic B field of $\sim 1\mu g$ *(microgauss)?*

Is the TeV proton traveling at relativistic or non-relativistic speed? We must first answer this question before we can calculate the gyroradius of the particle. To determine the answer to this question, we need to look at the Lorentz γ factor to see whether the value is greater than or equal to 1. If we knew the velocity of the proton, we could use the following definition to determine γ:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \tag{1}$$

where $\beta = v/c$.

Since we do not know the velocity, but instead are given the energy E, we can use the following relation for rest energy RE to find γ:

$$E = \gamma(RE) = \gamma(m_p c^2) \tag{2}$$

and therefore, we have

$$1\text{ TeV} = \gamma\left(\frac{938.272\ MeV}{c^2}\right)c^2$$

such that

$$\gamma = \frac{10^{12}\text{ eV}}{938.272 \times 10^6\text{ eV}} = 1065.789 \approx 1066$$

and the units cancel, as expected (since γ is dimensionless). We can now solve for the radius since we know the proton is relativistic, and can substitute the speed of light for the velocity. However, if you would like to know the exact value of the velocity, we can use Equation 1 and rearrange to solve for v:

$$v = \sqrt{c^2\left(1-\frac{1}{\gamma^2}\right)} = 298,999,868.4\text{ m/s}$$

and we can clearly see a substitution of $v = c$ is reasonable as an approximation.

In the case of a non-relativistic particle, $\gamma = 1$, and the equation for the gyroradius is:

$$r_g = \gamma \frac{m_p v}{zeB} = \frac{m_p v}{qB} \tag{3}$$

where z = number of protons and e is the elementary charge (since the particle is a proton, we can simply replace ze with q).

Inserting the value for γ and substituting $v = c$, we have the gyroradius for the relativistic 1 TeV proton in a 1 μg B field as:

$$r_g = \gamma \frac{m_p v}{zeB} = 1066 \left(\frac{m_p v}{qB} \right) = 3.326 \times 10^{13} \text{ m} \tag{4}$$

which we arrived at using the mass in kg for the proton, the velocity in m/s, the charge in C and the magnetic field in gauss units, which are $kg/C \cdot s$.

$$B = 1\ \mu g \times \frac{1\ g}{10^6\ g} \times \frac{10^{-4}\ kg/C \cdot s}{1\ g} = 10^{-10} \frac{kg}{C \cdot s}$$

Converting meters to parsecs:

$$3.326 \times 10^{13} \text{ m} \times \frac{3.24 \times 10^{-17} \text{ ps}}{1\text{m}} = 1.078 \times 10^{-3} \text{ ps}$$

Therefore, the gyroradius of a 1 TeV proton is:

$$r_g = 1.078 \times 10^{-3} \text{ ps}$$

2 References

1. Serway, Moses and Moyer. Modern Physics, 3rd Edition.
2. Longair, Malcolm. High Energy Astrophysics, 2nd Edition.