Relativistic Exercise 2

Krista Smith

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1 Fraction of TeV Neutrons that Survive the Earth's Atmosphere

What fraction of 1 TeV neutrons (from a pbh we are sensitive to) reach us before decaying?

The exponential decay equation can be used to determine the activity of a radioactive element after a certain period of time, or it can be used to determine the attenuation of light after passing through a certain material. We will use the equation here to determine the number of surviving neutrons (N) remaining after time t:

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

We are given that the time t is 5×10^6 seconds. We are also given that the lifetime, $\gamma\tau$, is 10^6 seconds, and that the energy E of the neutrons is 1 TeV. We can immediately find γ using the following relation to rest energy RE, as we did in the previous exercise:

$$E = \gamma(RE) = \gamma(m_n c^2) \tag{2}$$

$$1\ TeV = \gamma \left(\frac{939.565\ MeV}{c^2} \right) c^2$$

Rearranging the equation for γ, we have

$$\gamma = \frac{10^{12}\ eV}{939.565 \times 10^6\ eV} = 1064.322$$

And again, we see it is dimensionless. Next, we can find the value of the exponential decay constant λ using its relation to the lifetime τ:

$$\tau = \frac{1}{\lambda} \tag{3}$$

Since we have determined that the neutrons are traveling at relativistic speeds, we will need to include γ. Rearranging for λ:

$$\lambda = \frac{1}{\gamma\tau} = \frac{1}{10^6} = 10^{-6} \tag{4}$$

We do not know how many neutrons were present when they entered the upper atmosphere, but we don't need this information: We can pick the value (as long as we don't choose zero). Therefore, let $N_0 = 1$. Since there is still a factor of γ difference between the time the neutron takes to pass through the atmosphere in a rest frame near the neutron, compared with the time it would take as observed in a reference frame on Earth, we include this factor in the exponential equation:

$$N(t) = \gamma e^{-\lambda t} \tag{5}$$

and evaluating for $t = 5 \times 10^6$ we have

$$N = 1064.322e^{-10^{-6}(5\times10^6)} = 7.17135$$

In other words, if 1064 neutrons entered the upper atmosphere, only 7 of them would actually be detected at the surface of the Earth. The fraction of neutrons detected to the number of neutrons entering the atmosphere is expressed as:

$$\frac{7.17135}{1064.322} = 6.7379 \times 10^{-3}$$

Therefore, the fraction of 1 TeV neutrons that reach the surface of the Earth before decaying is 6.7379×10^{-3}.

2 References

1. Serway, Moses and Moyer. Modern Physics, 3rd Edition.
2. Longair, Malcolm. High Energy Astrophysics, 2nd Edition.