Deconvolution as Method of Cross-Check for fits of the Crab Nebula Spectrum

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Abstract

The Crab Nebula was first observed in 1054, but was only (relatively) recently identified as one of the brightest TeV photon sources in the sky in 1989.[1]. From the ground, gamma ray spectroscopy is very hard than from high in the atmosphere or even space where there is little to interact with gamma rays. The mean free path of a gamma ray in Earth's atmosphere is very low, making most incoming gamma ray's interact with particles in the atmosphere. This creates more particles that the initial gamma ray's energy is divided between, these particles decay or scatter into even more particles and so on. This amounts to what is known as an extensive air shower. This shower can be detected at 4000 meters above sea level (the Altitude of the HAWC observatory) by HAWC's vast array of water Cherenkov detectors with such precision that the incoming angle and energy of the initial particle can be reasonably reconstructed.[2] Observing sources in these very high energy bands, like the Crab is important, as it can inform theoretical models on a sources spectrum. Which can lead to insight on a sources properties and mechanisms. In the TeV band, the Crab Nebula is a standard candle whose spectrum is already well understood below 50 TeV[3], this implies that a spectral estimation tool used by The HAWC Observatory could be crosschecked with a deconvolution of data gathered from the Crab Nebula. In this case, the tool being checked is the neural network used in the HAWC collaboration, as well as many other fits to the Crab spectrum.

[1]H.E.S.S. Collaboration, "The Crab Nebula," HESS - The High Energy Stereoscopic System, October 1, 2004, accessed April 23, 2019, https://www.mpi-hd.mpg.de/hfm/HESS/pages/home/som/2004/10/.

[2]"HAWC." HAWC, HAWC Collaboration, 2011, www.hawc-observatory.org/.

[3]See footnote 1

Introduction

The Crab Nebula is the remnant of a supernova in the constellation Taurus. The nebula surrounding the neutron star at its center emits TeV gamma rays, when these gamma rays interact with the upper layers of earth's atmosphere they scatter into other particles and those will scatter and so on. This will produce a shower of sorts that will be detectable by the HAWC observatory's array of water Cherenkov detectors. When the average energy of a particle is around 80 MeV absorption will begin and the detectability shower will fade. If the HAWC observatory was at sea level, it would be extremely difficult to gather any meaningful data on very high energy sources as most- if not all, extensive air showers will have faded by the time their fronts reach sea level. Fortunately, the HAWC [4], or High-Altitude Water Cherenkov, Observatory is at a high enough altitude that its detectors are bombarded by many air showers long before they get the chance to fade. The array is composed of 300 tanks (these are the water Cherenkov Detectors), each 4 meters tall with a 7.3-meter diameter. The top of the Cherenkov detector is filled with water, while the lower portion contains 4 PMT's or photomultiplier tubes. These PMT's allow for the detection of these extensive air showers. When Particles from the shower strike the top of the detector they produce Cherenkov radiation, which is produced when particles travel faster than the speed of light in a particular medium (in this case, the speed of light in water is .75c). This is analogous to the sonic boom created by a jet exceeding the sound barrier. This 'photonic boom' emits light (mainly blue to ultraviolet light) which the PMT's are especially sensitive to. The PMT's can read the number of photons that strike them from the particles that initiated the Cherenkov radiation inside the tank. Since the energy of the particle that initiated the shower is divided between each particle of the shower, This data in conjunction with an estimation of how much of the shower actually interacted with the detector, can be used to estimate the energy of the particle that initiated the extensive air shower. Since these showers propagate through our atmosphere as an approximate planar front, the timing of the PMT hits can be used to extrapolate the direction at which the shower initiating particle came from. Using this data, it is possible to construct an energy spectrum of the Crab Nebula at these very high energies. One such tool is a neural network that is trained on simulated events, in which it is given the simulated data from the tanks and attempts to reconstruct the events energy, incoming angle, etc. At which point it will compare them to the true properties of the shower, and will make adjustments to make better estimations in the future. After the network is sufficiently trained, it can be subjected to real data, in the hopes that it will be able to extrapolate more accurate data. A cross check that this, and various other methods of the Crab energy estimation are producing spectra of the Crab as intended would be a simple deconvolution between the estimated energy of the particles from the Crab Nebula and how much of the shower from each particle landed on the detector.

[4] "HAWC." HAWC, HAWC Collaboration, 2011, www.hawc-observatory.org/.

1-Dimensional Deconvolution

The method by which this cross check is performed is centered on 2 quantities; the energy estimation of the initiating particle (here log(TeV)) and the Fraction hit, which is just a simple ratio between 0 and 1 that represents the fraction of the shower that landed within the detector. The first step is to construct a function that represents the accuracy of energy estimation as a function of energy. This can be accomplished by the assembly of a function that represents the ratio between the true count and the estimated count as a function of energy. Such a function could be represented as the ratio between 2 histograms. Since these 2 histograms, one representing the true count and the other representing the reconstructed count, will share the same binning scheme, it is possible to generate the points of this function, by calculating the ratio of each individual bin and storing it with that bin edge. This ratio between histograms can be turned into a scatter plot for which a function is fit, in this case, a second degree polynomial. The essence of this function is that within the domain of the original histograms it should yield the ratio between what the count truly is and what the count is observed to be at that energy, as shown in figure 1. The true count vs energy histogram is constructed from an uncut pool of events, while the reconstructed count vs energy histogram is constructed from the same pool but with cuts that remove the poorly reconstructed events.

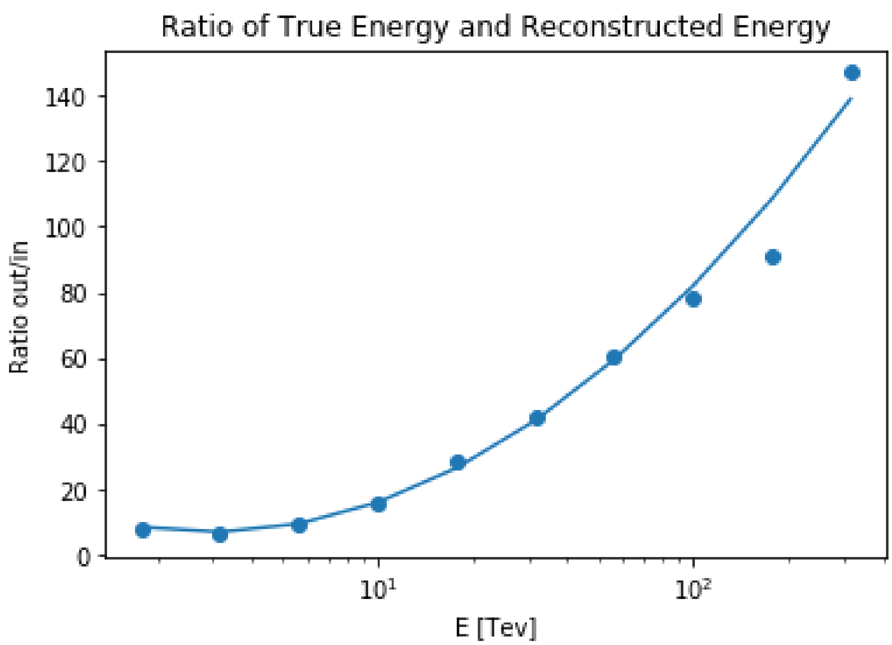

Figure 1: Ratio of True energy over Reconstructed energy as a function of energy

Each histogram has a binning scheme as show in table 1. This cut is necessary to eliminate more Hadron like events, which are background. The cuts based on the 'DelAngle' quantity or the difference between the reconstructed and the true incoming angle are necessary to simulate the detectors angular resolution in the PSF. The conventional representation for the Crab spectrum in this case is a plot of $\frac{dN}{dE}$ with units of $\mathrm{TeV}^{-1}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-2}$ vs energy with units log(TeV)Ȧ 2 dimensional histogram of counts from the Crab is constructed from detector response files; stored in each energy vs Fraction hit bin is the bin excess, or the counts observed subtracted by the expected counts due to background. This 2-dimensional histogram is then collapsed, i.e. all the Fraction hit bins within an energy bin are combined, and what is left is the photon count as a function of energy. Dividing each points Y value by the bin width will yield $\frac{dN}{dE}$. This function already represents the total count over the area of the detector calculated for the duration of the exposure time, which are obviously known quantities. in order to get units of flux per time, the function is divided by the throw area ($\pi 10^{10}$ cm^2) and the exposure time (69120000 s). This is the reconstructed flux as a function of energy, multiplying this function, shown in figure 2, by the ratio shown earlier (with units of True count/ Reconstructed count) should yield a better estimate of the true energy spectrum and can be used as a cross check to verify the neural network flux points of the Crab spectrum and various other fits.

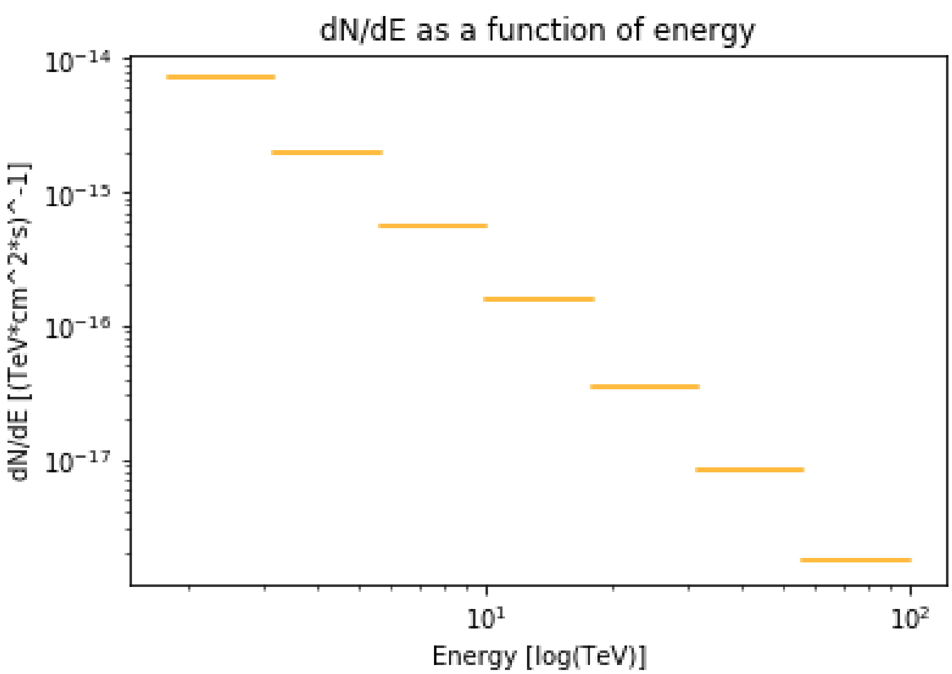

Figure 2: $\frac{dN}{dE}$ as a function of energy from data

Table 1: Energy Bin Edges used in the HAWC Collaboration

Bin Number	**Lower edge**(TeV)	**Upper edge** (TeV)
0	0.316	0.562
1	0.562	1.00
2	1.00	1.78
3	1.78	3.16
4	3.16	5.62
5	5.62	10.0
6	10.0	17.8
7	17.8	31.6
8	31.6	56.2
9	56.2	100
10	100	177

2-Dimensional Deconvolution

A more accurate method would be to calculate the ratio between the true and reconstructed energies in each energy-fraction hit bin, instead of calculating this ratio as simple a function of energy. This can be done by dividing 2 2-dimensional histograms that are populated using an events energy and Fraction hit, using the exact same energy scheme used for the 1 dimensional histograms above, but with a Fraction hit binning scheme shown in table 2. This time, instead of adding the bin excesses of each fraction hit bin within a given energy bin, each energy-fraction hit bin excess with be multiplied by the calculated ratio for that bin. Then the same process is applied; all the altered bin excesses are added up within their respective energy bin, then each point is divided by its energy bin width, the exposure time, and the throw area. This should produce a cleaner fit than the one-dimensional deconvolution and, therefore act as a better cross check. This analysis will use a fraction hit binning scheme shown in table 2. [5]

Table 2: Fraction Hit Bin Edges

Bin Number	**Lower edge**	**Upper Edge**
1	0.067	0.105
2	0.105	0.162
3	0.162	0.247
4	0.247	0.356
5	0.356	0.485
6	0.485	0.618
7	0.618	0.740
8	0.740	0.840
9	0.840	1

[5] A.U. Abeysekara Et Al. "OBSERVATION OF THE CRAB NEBULA WITH THE HAWC GAMMA-RAY OBSERVATORY." https://arxiv.org/pdf/1701.01778.pdf.

Results

These methods of deconvolution should be fit best to a log parabola, as suggested by the neural network as well as various other fits. Specifically it is assumed to be a log parabola characterized as such: $\frac{dN}{dE} = \phi_0(\frac{E}{E_0})^{(\alpha-\beta\log(\frac{E}{E_0}))}$.[6] according to fits from other collaborations. Shown in figure 3 is the ratio function (shown in figure 1) times the $\frac{dN}{dE}$ vs energy as well as the comparison between it and the spectrum given by the neural network, for which the former should be a cross check for.

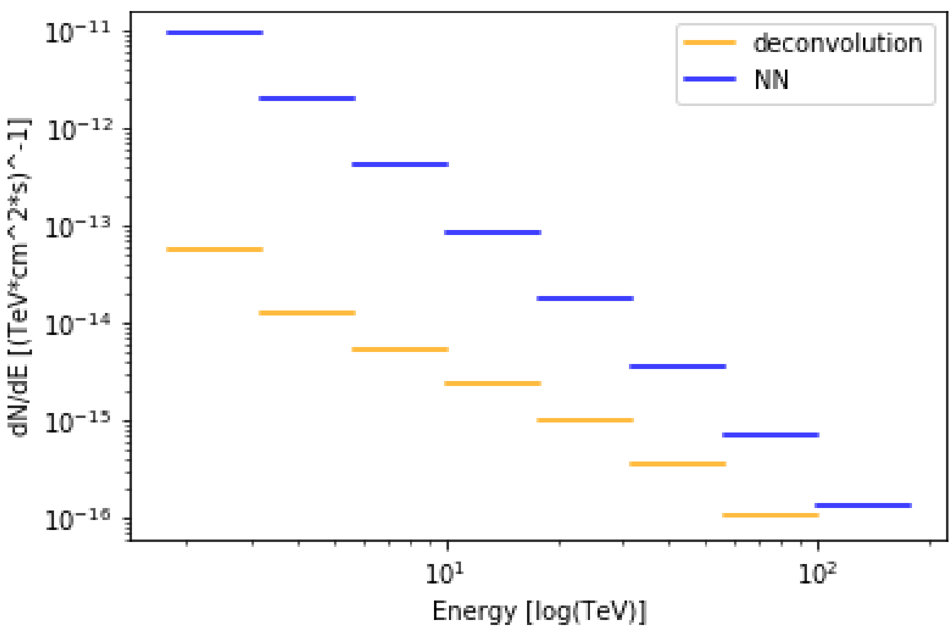

Figure 3: $\frac{dN}{dE}$ as a function of energy times ratio (orange) compared with the Neural Network spectrum (blue)

This shows that if this method of deconvolution was done correctly the method is too flawed to produce a viable cross check for the neural network. As shown in figure 4, the Neural network agrees with spectra produced by several other collaborations.

[6] Fraschetti, F., and M. Pohl. "Two-Zone Model for the Broadband Crab Nebula Spectrum: Microscopic Interpretation." EPJ Web of Conferences, 2017, doi:10.1051/epjconf/201713602009.

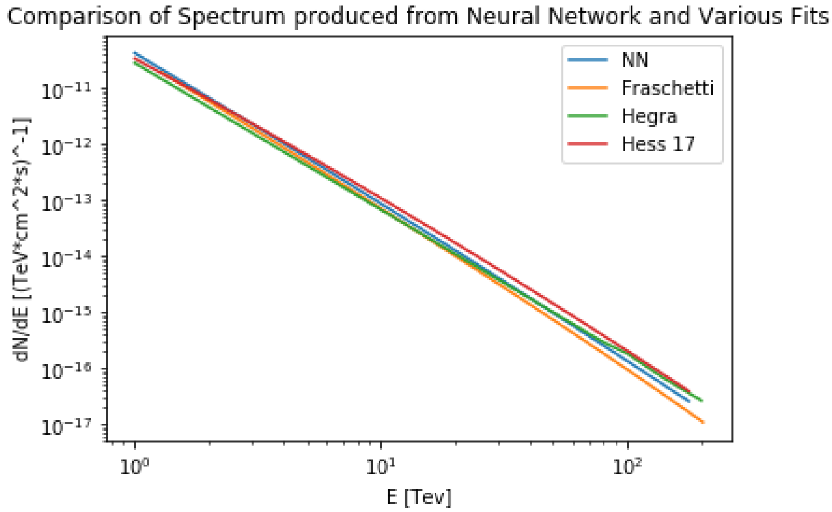

Figure 4: Neural Network compared with fits from other collaborations in functional form using parameters; $\alpha = 2.73$, $\beta = 0.06$, $\phi_0 = 2.31$, $E_0 = 7$ TeV

This increases the likelihood that instead there in an error in the data that went into the deconvolution, as most other possible sources of an error of this magnitude have been checked for and ruled out. Secondly the expected value of the deconvolution can be calculated if the function from Neural Network is integrated over each bin and displayed as a flat line, much like the histograms that the deconvolution are made of. This, as well as a comparison to the deconvolution, is shown in figure 5. The difference between The deoconvolution and this integration ranges from between 1 and 2 orders of magnitude.

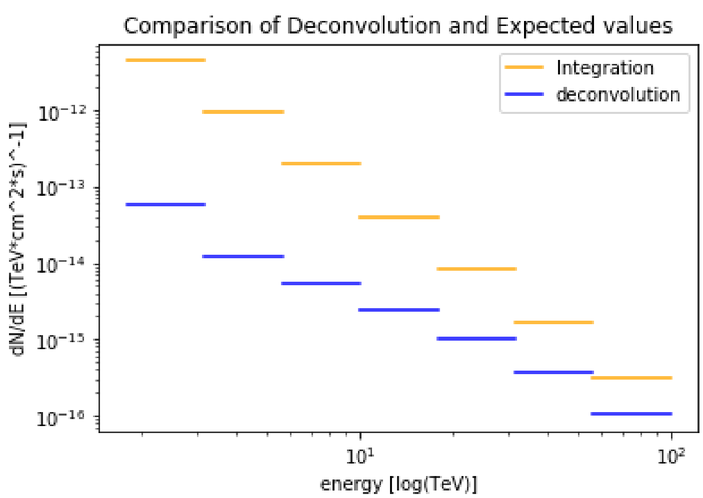

Figure 5: Deconvolution vs NN fit

This indicates that what this method of deconvolution has produced cannot be a valid result, the amount by which the difference varies between what is computed and what is expected implies that there is error in the computation or application of this method, as many possible errors in computation have been ruled out, and such errors should result in a predictable shift in the difference between the two curves.

Bibliography

[1] A.U. Abeysekara et al. "OBSERVATION OF THE CRAB NEBULA WITH THE HAWC GAMMA-RAY OBSERVATORY." 1701.01778.Pdf, HAWC Observatory, 17 Jan. 2017, arxiv.org/pdf/1701.01778.pdf.

[2] Fraschetti, F., and M. Pohl. "Two-Zone Model for the Broadband Crab Nebula Spectrum: Microscopic Interpretation."EPJ Web of Conferences, 2017, doi:10.1051/epjconf/201713602009.

[3] H.E.S.S. Collaboration, "The Crab Nebula," H.E.S.S. - The High Energy Stereoscopic System, October 1, 2004, accessed April 23, 2019 https://www.mpi-hd.mpg.de/hfm/HESS/pages/home/som/2004/10/.

[4] "HAWC." HAWC Collaboration, HAWC Observatory, 2011, accessed April 23, 2019 www.hawc-observatory.org/.